ASSIGNMENT SET – I

Mathematics: Semester-IV

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-402

Paper: Fuzzy Mathematics with Applications and Magneto Hydro Dynamics

Unit I: **Fuzzy Mathematics with Applications**

1	**a.** Evaluate the following: $\mathbf{2(5,6,8,12)+3(-1,3,4)-5(-3,2)+8}$. **b.** State Zadeh's Extension Principle. **c.** Let $\tilde{A}=\{(-2,0.45),\ (-1,0.50),\ (0,0.80),\ (1,\ 1),\ (2,\ 0.40)\}$ and $f(x)=x^2$. Find $f(\tilde{A})$. **d.** If $\tilde{A}$ = "real number considerably larger than 10" where, $$\mu_{\tilde{A}}(x)=\begin{cases}0, & x\le 10\\ (1+(x-10)^{-2})^{-1}, & x>10\end{cases}$$ Find A_α (α-level set) when $\alpha=0.50$. **e.** State Bellman and Zadeh's principle. **f.** Define a fuzzy multi-objective linear programming problem in general form. **g.** Discuss fuzzy sets concept with proper example. **h.** What is the difference between randomness and fuzziness?	Each question carries 2 marks

	i.	Prove that D'Morgan's law is true for fuzzy sets.	
2	**a.**	Show that for interval numbers distributive law does not hold in general.	Each question carries 4 marks
	b.	Using addition rule for fuzzy numbers, prove that $$(3,4,5)+(4,6,8)=(7,10,13)$$	
	c.	If $\tilde{A}\tilde{Y}=\tilde{B}$ be a fuzzy equation, find the solution $\tilde{Y}$ such that the membership of $\tilde{A}$ and $\tilde{B}$ are as follows: $$\mu_{\tilde{A}}(x)=\begin{cases}0, & x\le 3 \text{ and } x>5\\ x-3, & 3<x\le 4\\ 5-x, & 4<x\le 5\end{cases}$$ $$\mu_{\tilde{B}}(x)=\begin{cases}0, & x\le 12 \text{ and } x>32\\ \dfrac{x-12}{8}, & 12<x\le 20\\ \dfrac{32-x}{12}, & 20<x\le 32.\end{cases}$$	
	d.	Define LPP with Fuzzy Parameters and considering the fuzzy parameters as triangular fuzzy numbers, obtain its deterministic form step by step.	
	e.	Discuss the Verdegay's approach to solve fuzzy LPP.	
	f.	Write one of the methodologies to find the deterministic form of a 1st order linear fuzzy differential equation considering it as an initial value problem.	
	g.	State Zadeh's Extension principle. Use it to find fuzzy set $\tilde{B}=f(\tilde{A})$, where $f(x)=x^2-2$ and $\tilde{A}=\{(-3,0.8),(-2,0.5),(-1,0.3),(0,0.2),\ (1,0.6),\ (2,0.8),(3,0.9)\}$.	
	h.	Prove that the distributive properties of fuzzy set over standard union and intersection.	
	i.	Using addition rules of fuzzy numbers, show that 5+3=8 for real numbers.	
	j.	Let A=[a_1, a_2] and B=[b_1, b_2] be two interval numbers. Find A.B if (i) $a_1<0$, $a_2>0$, $b_1<0$ and $b_2>0$. (ii) $a_1>0$, $a_2<0$, $b_1<0$ and $b_2>0$.	
	k.	Let $\tilde{A}=[0,2,5]$ and $\tilde{B}=[3,4,6]$ be two triangular fuzzy numbers. Find $\tilde{A}\cup\tilde{B}$ and $\tilde{A}\cap\tilde{B}$.	
	l.	Simplify the following fuzzy expressions:	

	(i) $8[-4,0,1,3]-5[-3,1,7]+3[-10,5]-11$ (ii) $[-1,1,7]-4[-1,5]+17$	
3	**a.** Let, $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ be three fuzzy sets of a universal set X. The difference of $\tilde{A}$ and $\tilde{B}$ is defined by $\tilde{A}-\tilde{B}=\tilde{A}\cap\tilde{B}'$; and the symmetric difference of $\tilde{A}$ and $\tilde{B}$ is defined by $\tilde{A}\Delta\tilde{B}=(\tilde{A}-\tilde{B})\cup(\tilde{B}-\tilde{A})$. Prove that $(\tilde{A}\Delta\tilde{B})\Delta\tilde{C}=\tilde{A}\Delta(\tilde{B}\Delta\tilde{C})$. **b.** Consider the LPP-Model $Maximize\ z=2x_1+x_2$ $Such\ that\quad x_1 \lesssim 3$ $x_1+x_2 \lesssim 4$ $5x_1+x_2 \lesssim 3$ $x_1,x_2 \geq 0.$ The "tolerance intervals" of the constraints are $p_1=6$, $p_2=4, p_3=2$. Using Werner's method find its solution. **c.** Let $\tilde{A}$ and $\tilde{B}$be two fuzzy numbers whose membership are given by: $\mu_{\tilde{A}}(x)=\begin{cases}(x+2)/2, & -2<x\leq 0\\ (2-x)/2, & 0<x<2\\ 0, & otherwise\end{cases}$ $\mu_{\tilde{B}}(x)=\begin{cases}(x-2)/2, & 2<x\leq 4\\ (6-x)/2, & 4<x\leq 6\\ 0, & otherwise.\end{cases}$ Calculate the fuzzy number $\tilde{A}+\tilde{B}$. **d.** Assume that a company makes two products. Product P_1 has a \$0.40 per unit profit and product P_2 has a \$0.30 per unit profit. Each unit of product P_1 requires twice as many labour hours as each product P_2. The total available labour-hours are at least 500 hours per day, and may possibly be extended to 600 hours per day, due to special arrangements for overtime work. The supply of material is at least sufficient for 400 units of both products, P_1 and P_2, per day, but may possibly be extended to 500 units per day according to previous experience. Formulate the fuzzy LPP and solve it to answer "How many units of products P_1 and P_2 should be made per day to maximize	Each question carries 8 marks

the total profit?"

e. Prove that $[a_1, b_1, c_1, d_1] + [a_2, b_2, c_2, d_2] = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]$, where $[a, b, c, d]$ is a trapizoidal fuzzy number.

f. Explain Zimmermann's method to convert fuzzy LPP to crisp LPP.

g. Using Werner's method, find the crisp LPP corresponding to the following fuzzy LPP as

$$\begin{aligned} &Max \quad Z = x_1 + x_2 \\ &Subject \quad to \\ &-x_1 + 3x_2 \le 21 \quad to \quad 23 \\ &x_1 + 3x_2 \le 25 \quad to \quad 27 \\ &4x_1 + 3x_2 \le 45 \quad to \quad 50 \\ &x_1, x_2 \ge 0 \end{aligned}$$

h. Using Verdegay's method convert the following fuzzy LPP to corresponding crisp LPP

$$\begin{aligned} Max \quad & Z = x_1 + 2x_2 \\ s.t. \quad & -x_1 + 5x_2 \lesssim 9 \\ & 4x_1 + 3x_2 \lesssim 17 \\ & 3x_1 + 2x_2 \lesssim 14 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Given that the tolerance levels p_i of constraints are as $p_1 = 3, p_2 = 4, p_3 = 7$.

Unit II: **Magneto Hydro Dynamics**

1. Write Navier stokes equation of motion.
2. Define the term magnetic diffusivity.
3. Write down the working procedure of 'magneto-fluid-dynamics (MFD) submarines'.
4. Define Reynolds number and explain its significance.
5. Write down the statement of Alfven's theorem
6. Define MFD submarines.
7. Write the Maxwell equations for electrostatics.
8. Define Alfven velocity and Alfven waves.
9. Define Magnetic Mach number.
10. Define magnetic Prandtl number.
11. Write a short note on 'Hall effect' for the mageto-hydrodynamics flow
12. Write down the Maxwell's electromagnetic field equations of moving media.
13. Define the terms 'drift velocity' and 'magnetic diffusivity'.
14. Define Magnetic Mach number.
15. Define MHD Couette flow.
16. State and prove Alfaven's theorem.
17. Derive energy of the magnetostatic field.
18. Solve th problem of MHD coquette flow. Dirichlet problem.
19. Derive the velocity expression of MHD flow if it passes through two parallel plates.
20. Show that magnetic body force per unit volume for a conducting fluid in a magnetic field is equivalent to a tension per unit area along the lines of force, together with a hydrostatic pressure
21. Show that the charge decays very rapidly in an exponential manner at any point within a conducting fluid at rest.
22. For a conducting fluid of magnetic field, show that the magnetic body force per unit volume, i,e $\mu(\nabla \times H) \times H$ is equivalent to a tension μH^2 per unit area along the lines of force, together with a hydrostatic pressure $\frac{1}{2}\mu H^2$ where symbols have their usual meaning.

23. Show that the tangential component of the magnetic field intensity is discontinuous across the surface.
24. Show that magnetic body force per unit volume for a conducting fluid in a magnetic field is equivalent to a tension per unit area along the lines of force together with a hydrostatic pressure
25. Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis.
26. Show that the electrostatic potential over an isorotational surface is constant
27. Write down the basic equations of magneto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.
28. Prove that for a conducting liquid, the flux of the magnetic field through a closed circuit of the fluid particles moving along with the fluid is constant for all time
29. A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = -L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$ (upper). Assume that a uniform magnetic field *H0* acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field.
30. A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = -L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$ (upper). Assume that a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field.
31. Define magnetic energy and further, find the rate of change of magnetic energy in magneto-hydrodynamic.
32. Show that the tangential component of the magnetic field intensity is discontinuous across the surface.
33. Show that the charge decays very rapidly in an exponential manner at any point within a conducting fluid at rest.
34. Show that the electrostatic potential over an isorotational surface is constant.
35. Define MHD Couette flow. Derive the velocity expression of MHD flow if it passes through two parallel plates.

36. Write down the basic equations of mageto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.

37. Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis.

38. Define the terms Alfven's velocity and Alfven's waves. Hence, derive the speed of propagation is $\sqrt{c^2 + V_A{}^2}$ for magneto hydrodynamic wave, where symbols have their usual meaning.

Define Couette flow. Give the mathematical formulation of mageto-hydrodynamic Couette flow and derive its velocity and magnetic field expression.

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